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ABSTRACT

Facets of problem solving instruction (PSI) were studied in German mathematics classrooms. Nine different facets of PSI were identified from the research literature, and a video rating form was developed to assess these facets and teacher effectiveness. Videotaped mathematics lessons (n=147) for German eighth graders from the sample for the Third International Mathematics and Science Study (TIMSS) were rated, each by 2 raters. Some of the PSI facets were seen very rarely. Results from factor analyses show that the facets have a three-dimensional structure. Although these facets were not directly related to achievement and learning, they were related to lesson characteristics and aspects of classroom discourse. Correlations with the student ratings of frequencies of various instructional activities also support the validity of the PSI facets. A reasonable degree of teacher effectiveness can be regarded as a precondition for a constructivist problem-solving approach. (Contains 5 tables, 2 figures, and 10 references.) (SLD)

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Identifying Facets of Problem Solving in Mathematics Instruction

Paper presented at the AERA Annual Meeting

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Introduction: Towards a Problem-Solving Instruction

This talk is focussed on the identification and evaluation of facets of problem solving instruction (PSI) in the natural setting of German Mathematics classrooms. It is supposed to give an impression of the way our research team at the Berlin Max-Planck Institute for Human Development is trying to link the TIMS-Study and the TIMS-Video-Study to create a network of explanatory power that goes beyond the international achievement horserace.

What is PS?

PSI can be regarded as the critical joint between instructional research and modern learning theories of cognitive psychology. PSI goes beyond simply confronting students with well-defined addition problems, multiplication problems, fraction problems or word problems as it is done in any Mathematics classroom.

The specific use of the term "problem" in the context of PSI needs to be clarified: Its early roots can be identified in the philosophical works of Dewey, who speaks of problems as "forked-road-situations" that call on human curiosity as a natural resource. Problems can be roughly described as "nonroutine questions of some complexity". They include a non-desired starting situation and a desired goal with the actual means to the specific end being initially unknown to the person faced with it. A problem in itself carries a gap, a disturbance or an incongruity which stands between the question and it's solution. These gaps that make a problem "nonroutine" or "ill-structured" are supposed to have a motivating and challenging function on the individuals confronted with them.

Schoenfeld (1989) describes "problem" in the context of Mathematics instruction as "a task (a) in which the student is interested and engaged and for which he wishes to obtain a resolution, and (b) for which the student does not have a readily accessible Mathematical means by which to achieve that resolution" (p. 88).

Different Research Frameworks and Conceptualizations of PSI

This is not the place to fully acknowledge the body of research that has been accumulated on PSI. The following quotations are meant to illustrate how different researchers stress different facets of PSI in Mathematics. Problem solving instruction can be differently understood from different theoretical approaches.

Hiebert, Carpenter, Fennema et al. (1996) are relating to the works of Dewey and postulate to incorporate problem solving in the classroom. "Students should be allowed to make the subject problematic. We argue that this single principle captures what is essential for instructional practice." Palincsar (1998) stresses the role of the teacher as a mediator, a promoter of discussion among pupils in the context of a constructivistically oriented instruction. "The crucial role that the teacher plays in promoting the co-construction of knowledge in classrooms was also demonstrated in the research [...] (on) the dynamic role of the teacher in guiding classroom discussions in the context of Mathematical problem solving." The theoretical works on PSI found their way into the Mathematics curriculum in the U.S.: In the 'curriculum and evaluation standards for school Mathematics' released by the National Council of Teachers of Mathematics (NCTM, 1989) PSI is given a prominent role: "Problem solving is the process by which students experience the power and usefulness of mathematics in the world around them. It is also a method of inquiry and application, interwoven throughout the Standards to provide a consistent context for learning and applying mathematics. Problem situations can establish a "need to know" and foster the motivation for the development of concepts." The current draft version 'principles and standards for school Mathematics' ('Standards 2000'; NCTM, 1998) puts an even larger weight on the implementation of PSI in Mathematics classrooms.

The NCTM Guidelines influenced educational practice not only in the U.S. In the 'Open-Ended Approach' Japanese educators blended central elements of the NCTM approach with Japanese ideas of PSI (Becker & Shimada, 1997). Results from the TIMSS-Video-Study show that "Japanese Mathematics teachers systematically use alternative representations and student-developed solution methods to reveal the Mathematical problem" (Baumert, Lehmann et al., 1997; authors translation). Nohda (1995) underlines the importance of confronting students with open-ended Mathematical problems. These are problems that allow the use of alternative solution methods that can lead to correct solutions on different levels of complexity: "In Japan, new approaches to problem solving are being emphasized in school Mathematics. [...] (The teaching approach using "open-ended problems") has been shown to be the most effective in fostering both students' Mathematical thinking and students' motivation toward Mathematics."

All these researchers unanimously promote problem solving as a necessary, if not crucial component of a cognitively demanding instruction. But the exact nature of the relation between (1) problem solving as an overt, organized classroom activity, (2) problem solving as a type of cognitive processing and (3) problem solving as a more or less generalized competency often is not made clear.

Research Questions

Empirical research from the different PSI frameworks described above has been focussed on experimental small group settings and interpretative microanalyses of the learning process. The question to what degree facets of PSI apply in the natural environments of everyday Mathematics classrooms and in what way these facets interact with the other features of instructional quality has been seldomly addressed. Goal of this study is to examine the nature of problem solving as a construct in instructional research using high inferential video ratings for nine different facets of PSI which where derived from research literature. In our exploration of problem solving as a construct we try to answer the following questions.

Is it possible to define different facets of the construct PSI and to empirically identify them? How frequently do the different facets of Problem Solving Instruction apply in the natural setting of everyday German Mathematics classrooms? Can the different facets be integrated into a unidimensional instructional characteristic? Is there a unique specific problem solving variance which is distinct from other characteristics of Teacher Effectiveness? Can some elements of Teacher Effectiveness be identified as meaningful conditions for a Problem Solving Instruction? Does Problem Solving Instruction directly improve learning in Mathematics? In what way are the facets of problem solving instruction related to classroom discourse and structural characteristics of instruction? Do the external video ratings based on a small video sample correspond to the students' general view of instruction?

Study Design

Nine facets of problem solving instruction are derived from different approaches in research literature. To assess these nine PS facets a video rating form is developed. This rating form is used in combination with a classroom observation inventory developed by Helmke, Schrader and Weinert (Weinert & Helmke, 1997), in which classic constructs of instructional quality such as teacher control, clarity of rules, organization and time-on-task are measured. The videotaped 8th grade Mathematics lessons rated, are part of the German TIMS-Video Sample (Third International Mathematics and Science Study; Beaton et al., 1996; Baumert et al., 1997). Since the videotaped classes also participated in the TIMSS-Main study, this study provides us with curricular valid achievement scores and video recordings of up to three consecutive lessons. The resulting ratings of instruction are related to reference data from TIMSS, including student achievement and motivation (TIMSS and German longitudinal enhancement), low inference codes (TIMS-Video) and student perceptions of instruction (TIMSS-Student Background Questionnaire).

Nine Facets of Problem Solving Instruction

The following nine facets are meant to represent possible realization of problem solving in Mathematics instruction:

1. Problem Solving Processes
   (Reference: traditional research on problem solving)
2. Focusing
   (Reference: Japanese open-ended approach)
3. Teacher as a Mediator
   (Reference: Japanese open-ended approach; constructivism)
4. Openness of problems and solutions
   (Reference: Japanese open-ended approach; constructivism)
5. Anchoring in student knowledge
   (Reference: constructivism)
6. Student Discussion
   (Reference: constructivism)
7. Student Cooperation
   (Reference: constructivism; Situated Learning)
8. Multiple Contexts
   (Reference: Situated Learning)
9. Authentic real-world embedding
   (Reference: Situated Learning)

The facet "PS Processes" is derived from traditional psychological research on problem solving. In this facet we want to assess to what degree an instruction can be seen as an explicit goal-directed, multi-step PS process including evaluation and feed-back steps. "Focussing" describes the teacher's concentrating on main goals or questions in the course of classroom interaction. The facet "Teacher as a Mediator" assesses whether the teacher is functioning as a mediator or promoter of the exchange of students ideas in discussions. The facet "Openness of Tasks and Solution Approaches" asks whether there are more than one possible correct solutions for the tasks and problems given. "Anchoring in Student Knowledge" assesses the degree to which a teacher tries to connect the given problems to Mathematical knowledge the students learned in previous lessons or courses. The facet "Student Discussion" asks whether the students are given the chance to negotiate Mathematical concepts and meanings. Another facet, "Student Cooperation" is focussed on whether students can work together in small groups on the given problems. "Multiple Contexts" refers to whether the teacher presents problems in more than one embedding, e.g. in different Mathematical contexts or using different media. The facet "Authentic Real-World Embedding" assesses the use of authentic problem situations or real-world problems. The following table (Table 1) shows some examples from our observation instrument.

Table 1: Example for the Problem Solving Item "Teacher as a Mediator"

	Does fully apply	Does partially apply	Does partially not aplly	Does absolutely not apply
3. Teacher as a mediator (Reference: Japanese open-ended approach; constructivism)	❒	❒	❒	❒
The teacher gives time to the students to develop ideas and find answers.				
The teacher puts the ideas of the students in relation to the context of the lesson.				
The teacher puts the contributions of the students in relation to each other.				
The teacher supports the students in formulating their ideas.				
If a student formulates incomplete and unclear ideas, the teacher investigates them without judging the students performance.				
The teacher doesn't correct immediately every mistake.				

Technical Details

The German TIMSS Video-Sample consist of 1st Lessons from 82 classes, of which 34 have 2nd lessons of which 31 have 3rd lessons, resulting in a total number of 147 lessons. Each lesson was judged by two raters, so that 294 ratings were completed. The rating instrument includes 9 Items on Problem Solving and 36 classical Teacher Effectiveness Items. The video ratings were completed by trained raters. Training involved thorough discussion of the rating instrument and an extensive practicing phase in which several demo lessons where rated. Each video was judged independently by two raters. Each possible pair of raters saw one third of the lessons with each trisection of the sample balanced according to type of school. After watching a complete videotaped lesson the raters indicated presence or absence for each of the 45 items on a 4-point rating scale (1 'does not apply' to 4 'does fully apply'). Aggregated item scores for each video were computed on the basis of averaged ratings by the two independent judges. Averaged weighted kappa between raters was .77.

Results

Our first research question was how frequently the different facets of Problem Solving Instruction apply in the natural setting of everyday German Mathematics classrooms. Figure 1 shows that six of the nine facets can be regarded as rather rare events in German Mathematics instruction. Only the top three facets, PS Processes, Focussing and Teacher as a Mediator have approximately normal distributions.

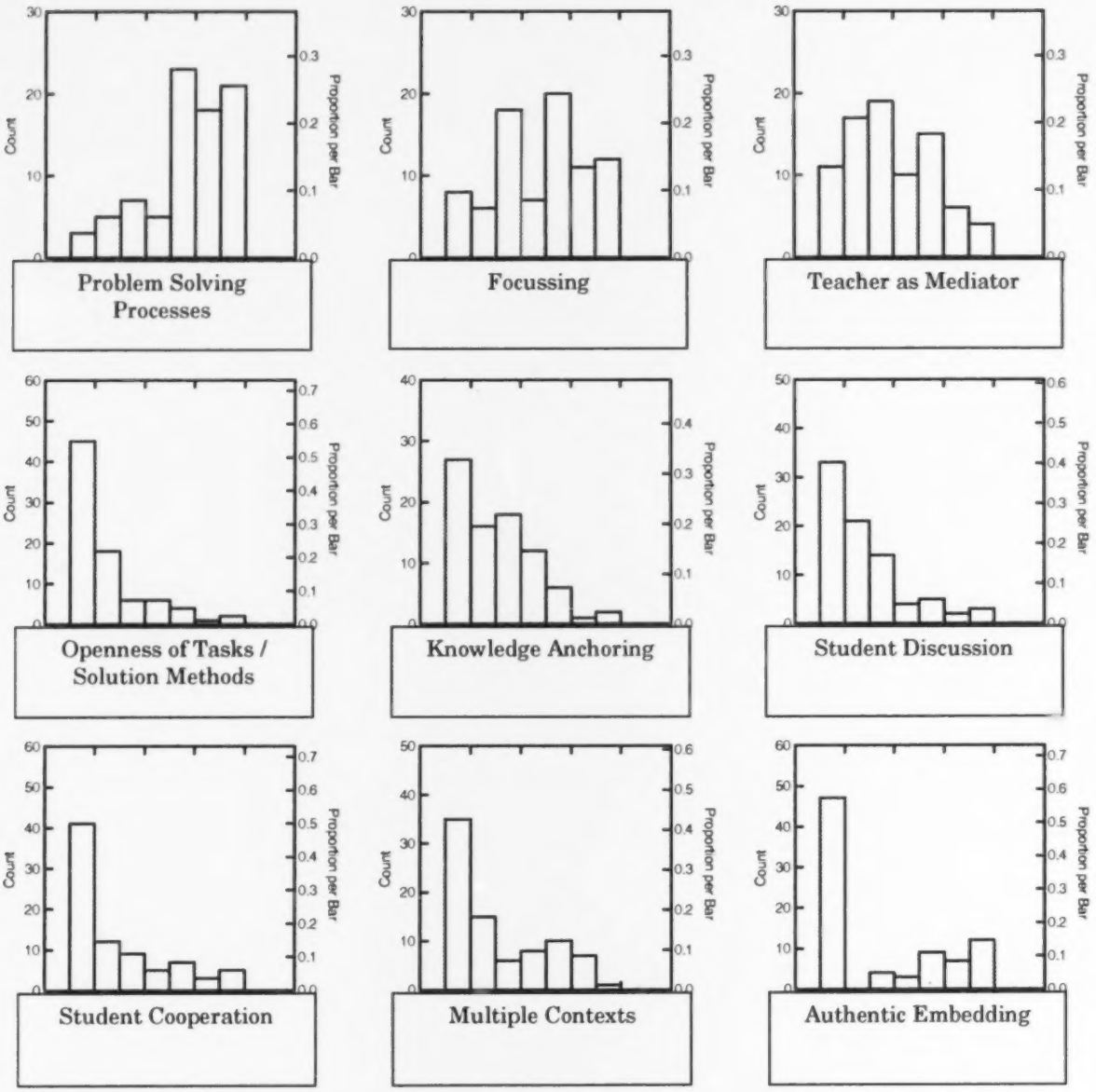

Figure 1: Distributions of the nine Problem Solving Facets

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Can the 9 facets be considered as realizations of a unidimensional construct? Table 2 shows the resulting loading matrix of a varimax rotated principal component analysis.

Table 2: Varimax-rotated Principal Component Analysis of the nine PS Facets

	Factors		
	1	2	3
	"Constructivist Instruction"	*"Problem Processing"*	*"Real-World Embedding"*
	Argumentative reflective interaction	Goal-oriented and focussed solution approach	Problems in multiple natural contexts
Student Discussion	0,84		
Teacher as Mediator	0,84		
Openness	0,81		
Student Cooperation	0,80		
Focusedness		0,92	
Problem Solving Process		0,64	0,45
Knowledge Anchoring	0,51	0,58	
Authentic Embedding			0,92
Multiple Contexts			0,75

Loadings under 0,40 are omitted

PSI Facets

The analysis brought up three factors with eigenvalues greater 1. Looking at the loading coefficients, three main components are separated:

(I) argumentative and reflective interaction ("Constructivist Instruction")

(II) goal-oriented and focussed solution approaches ("Problem Processing")

(III) problems in multiple natural contexts ("Real-World Embedding")

The second component is a little less distinct than the other two considering the side loadings. We cannot say that the facets represent a single unidimensional construct.

How do the problem solving facets relate to classic dimensions of teacher effectiveness? The following table 3 shows another factor analysis in which we analyzed the PS facets together with the 36 other observation items.

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Table 3: Factor Analysis Varimax-rotated PCA Effectiveness Dimensions and PSI Facets

	Effective Instruction	**"Constructivist Instruction**	Structure / Clarity	Seatwork factor	Low-level fostering/ coping	Quality of interaction	**"Real-World Embedding"**	socio-emotional Climate	Achievement orientation	
	1	2	3	4	5	6	7	8	9	10
ORGANISATION	0,87									
CONTROL	0,86									
RULECLARITY	0,84									
TIMEONTASK	0,81									
SCONTINGENCY	0,78									
ACH.-CLIMATE	0,75									
SPEED	0,56				-0,44					
CONCISE	0,54		0,41							
DISCUSSION		**0,80**								
MEDIATOR		**0,77**								
KNOWLEDGE		**0,68**								
OPENNESS		**0,66**								
COOPERATION		**0,57**			0,45					
ENGAGE		0,45								
PREVIEWS			0,81							
FOCUSED			**0,79**							
CUES			0,72							
STRUCTURE	0,44		0,63							
SEQUAL				0,88						
SACVTIVE				0,85						
SFOSTER				0,74						
FOSTER					0,82					
INDIVIDULIZE					0,76					
REACTION						0,76				
SDISCRETE						0,75				
RELATION		0,43				0,59				
AUTHENTIC							**0,82**			
MULTICONTEXT							**0,64**			
ACCAFFECT								0,81		
ACCPRIVAT								0,81		
TEACHERFOCUS								-0,37		
DIFFICULTY									0,80	
PROBLEMPROC		**0,42**							**0,44**	
VARIABILITY									-0,41	
NORM/VALUE										0,64
LEARNTECH										0,56

Results indicate that the second and the seventh factor represent unique problem solving variance, which is not already assessed in the teaching effectiveness dimensions. These two factors correspond to the dimensions "Constructivist Instruction" and "Real - World Embedding", which emerged in the first analysis. The dimension "PS Processes" breaks up and is partially represented in the factor called Structure/Clarity.

So is there no relation between instructional effectiveness and a PS teaching approach? The answer is no. The following figure (Figure 2) shows that there are interesting relations, which are not depicted in correlations or factor analyses.

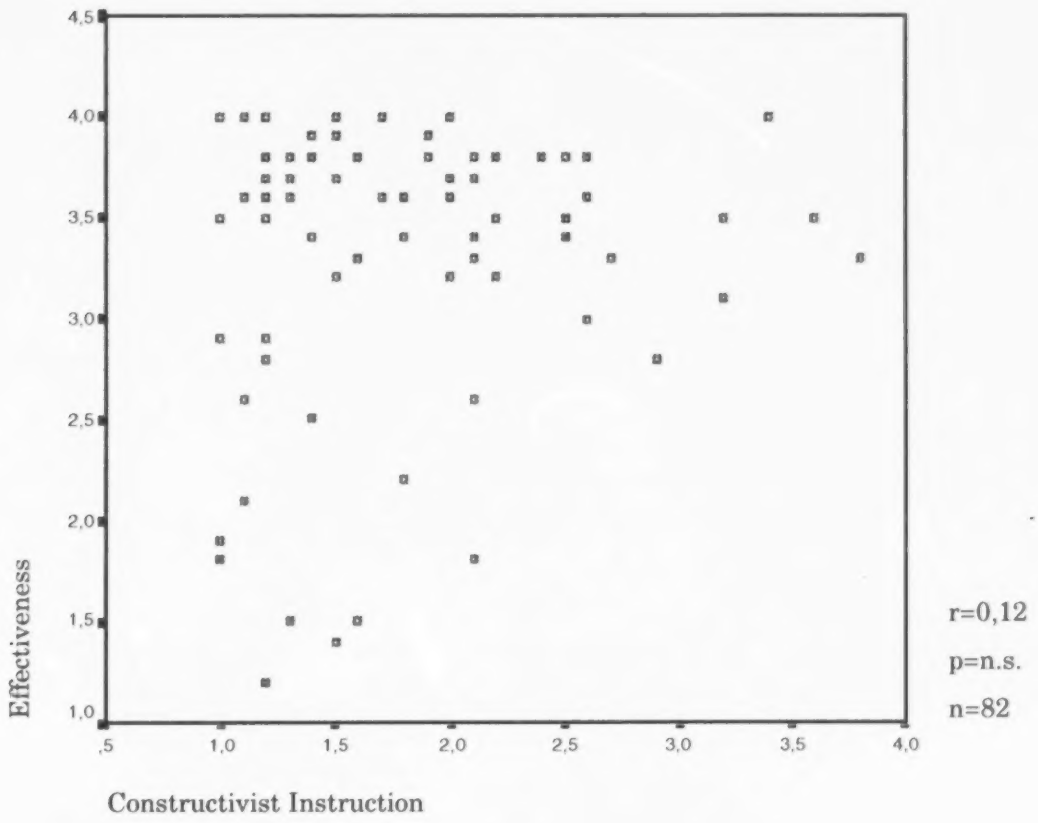

Figure 2: Constructivist Instruction and Effectiveness

This scatter plot displays the dimension "Constructivist Instruction" plotted against the dimension Effectiveness, a global measure consisting of control, speed, rule clarity, organization and time-on-task. The empty field shows that there are no ineffective lessons in which constructivist instruction can be found. A reasonable level of effectivity may be regarded as a precondition for the realization of student discussion and cooperation in a constructivist learning environment.

Does Problem Solving Instruction directly improve learning in Mathematics? The German longitudinal enhancement of the TIMS study enables us to look at both achievement and learning in Mathematics.

Table 4: Correlations with Achievement, Learning and Interest Growth

	Achievement		Interest Growth
	TIMSS Score (n=82 classrooms)	Growth 7 to 8 (n=81 classrooms)	Mathematics as a subject (n=67 classrooms)
PS Constructivist Instruction	,	,	**0,27 ***
PS Problem Processing	,	,	,
PS Real-World Embedding	,	,	,
Clarity of Rules	0,20	,	,
Organisation	0,16	,	,
Control	0,17	,	,
Time-On-Task	**0,27 ***	0,15	,
Speed	**0,33 ****	**0,23 ***	,
Teacher Focusedness	0,16	,	-0,22

* $p < .05$, ** $p < .01$; coefficients below + / - 0.15 are omitted (,)

A look at table 4 shows that PS Instruction assessed by our three dimensions neither related to the achievement level at grade 8 (the class average of the TIMSS score) nor does it influence the learning form grade 7 to grade 8. In comparison the teacher effectiveness dimensions prove as being effective in fostering learning. On the other hand there may be an indirect influence of problem solving. The dimension Constructivist Instruction is positively correlated to the development of the students interest in Mathematics as a subject.

In what way are the facets of problem solving instruction related to classroom discourse and structural characteristics of instruction? The TIMSS-Video Study provides us with a lot of information on low-inference characteristics of instruction, classroom interaction and classroom discourse. Significant relations between the PS dimensions and such characteristics are displayed in the following transparency.

Constructivist Instruction is positively related to sharing, group seatwork and tasks that involve inventive thinking during seatwork phases. Constructivist instruction goes along with more and longer utterances by the students and with more utterances in which a student or the teacher evaluates a preceding utterance. It is negatively related to chalkboard use and to "non-cooperative" individual seatwork.

Lessons high in Problem Processing show more interactions in the form of "one task / one situation" and more teacher utterances characterized as "information". In such lessons fewer interactions in the form of "one task / multiple situations" and fewer discipline-related teacher utterances are found.

Real-World Embedding is positively related to the use of manipulatives or models. Lessons high on this dimension show more student generated solutions, more alternative solutions and a higher amount of teacher talk with a demonstration purpose. On the other hand these lessons are negatively related to the complexity of topic taught.

Do the external video ratings based on a small video sample correspond to the students' general view of instruction? To answer this question we looked at the correlations of the PS dimensions with a set of items from the TIMSS student background questionnaire, in which the students were asked "How often do the following things happen in your Mathematics classroom?".

Table 5: Problem Solving and Student Perceptions

How often do the following things happen in your mathematics classroom?

TIMSS Background Questionnaire (response format: always often - pretty often - once in a while - never)

N=82	PS Constructivist Instruction	PS Problem Processing	PS Real-World Embedding
teacher shows us, how to solve mathemat. problems	**0,29****	**0,24***	**0,23***
copy notes from the board	,	,	-0,17
have a quiz or test	-0,17	,	,
work from worksheets on our own	,	,	,
work on mathematical projects	0,21	,	0,17
work in pairs or in small groups	**0,33****	0,19	**0,30****
solve problems with everyday life things	,	**0,28***	**0,26***
discuss homework	0,20	-0,17	,

* $p < .05$, ** $p < .01$; coefficients below + / - 0.15 are omitted (,)

Results in table 5 show that the item "The teacher shows us how to solve Mathematical problems." has significant positive correlations with all three PS Dimensions (and all nine PS facets). Furthermore the first and third dimensions, Constructivist Instruction and PS Processing, are positively related to the Items "work in pairs or small groups". PS Processing and Real World Embedding are positively related to the "solve problems with daily life things". The pattern of correlations shows that the video ratings are related to the students general perception of their Mathematics instruction.

Summary

To assess problem solving in Mathematics instruction nine high-inference observation items were derived from literature. These items were used in combination with a classical observation system of teacher effectiveness to rate the 8th grade Mathematics lessons from TIMSS. Some of the PS facets can be regarded as rare events. The results from factor analyses show that the facets have a 3-dimensional structure. The problem solving variance in the instructional behavior is specific and is not already represented in the classical features of instruction. A reasonable level of teaching effectiveness can be regarded as a precondition for a constructivist PS approach. The PS dimensions are not directly related to achievement and learning. A constructivist PS approach is related to interest growth in Mathematics. The PS facets are related to lesson characteristics and aspects of classroom discourse. Correlations with the student ratings of frequencies of various instructional activities also support the validity of the PS facets.

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